Test 3 Review Problems

1) Find the determinant of $\begin{bmatrix} 2 & 1 & 3 \\ 0 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

2) Find $\begin{vmatrix} -5 & 3 & 1 \\ 6 & 2 & 2 \\ 4 & 4 & 0 \end{vmatrix}$.

3) Find the determinant of $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 5 & 1 & 0 & 0 \\ 6 & 1 & 0 & 0 \end{bmatrix}$

4) Find the determinant of $A - xI_4$ where $A = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 3 & 4 \\ 5 & 1 & 0 & 0 \\ 6 & 1 & 0 & 0 \end{bmatrix}$.

5) Explain why $|A| = |A^t|$

6) $|A| = 6$. B is obtained from A by: (a) interchanging two rows, (b), multiplying the 4th row by 7, (c) interchanging another two rows, and (d) adding 5 copies of the 3rd row to the 5th row. Find $|B|$.

7) For a 3×3 matrix, show that $|cA| = c^3|A|$ where c is a scalar.

8) Find the eigenvalues and eigenvectors of $\begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix}$.

9) Find the characteristic polynomial of $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ -1 & 1 & 2 \end{bmatrix}$

10) Find the eigenspaces of $\begin{bmatrix} 3 & -1 & 0 \\ -1 & 3 & 0 \\ -1 & 1 & 2 \end{bmatrix}$

11) Find the eigenvectors of $\begin{bmatrix} 5 & 5 & 1 & 8 \\ 8 & 2 & 1 & 8 \\ -6 & 6 & -9 & 0 \\ -7 & -1 & -2 & -10 \end{bmatrix}$. (Hint: $-3, -6$, and -9 are eigenvalues)

12) Let $\beta_1 = \left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} \right\}$. What is $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}_{\beta_1}$ expressed in the standard basis?

13) What is $\begin{bmatrix}1\\0\\0\end{bmatrix}$ expressed in the basis β_1?

14) Is it possible to express $\begin{bmatrix}-1\\-1\\-1\end{bmatrix}_S$ in terms of β_1? If so, find it.

15) Let $\beta_2 = \left\{\begin{bmatrix}-1\\-3\\1\end{bmatrix}, \begin{bmatrix}1\\4\\-2\end{bmatrix}, \begin{bmatrix}-2\\-3\\-2\end{bmatrix}\right\}$. Find $[I_3]_{\beta_1}^{\beta_2}$, the change of basis matrix from β_1 to β_2.

16) Find $\begin{bmatrix}1\\2\\-1\end{bmatrix}_{\beta_1}$ in terms of β_2.

17) Find $\begin{bmatrix}0\\0\\-3\end{bmatrix}_{\beta_2}$ in terms of β_1.

18) Find $\begin{bmatrix}1\\2\\3\end{bmatrix}$ in terms of both β_1 and β_2. Graph all three of these on the same plane.

19) Diagonalize $\begin{bmatrix}-2 & 2\\0 & 0\end{bmatrix}$.

20) Diagonalize $A = \begin{bmatrix}4 & -1 & -2\\-6 & 3 & 4\\8 & -2 & -4\end{bmatrix}$.

21) Find A^2

22) Find A^{20}

23) Find A^{200}

24) Diagonalize $\begin{bmatrix}1 & 0 & 0 & 0\\1 & 2 & 0 & 0\\1 & 0 & 3 & 0\\-1 & 0 & 1 & 7\end{bmatrix}$

25) A diagonalizable 7×7 matrix A has eigenvalues $2, 3, 4$ with 3 having multiplicity 5. Show that $A - 3I_7$ has exactly 2 linearly independent rows.